Interest Rates Are Not Mean Reversionary

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Introduction

In many of the well known interest rate models there is an assumption of some variant of mean reversionary behaviour. This can be seen in formulae that drags interest rates towards a fixed mean.

Table 1: Well Known Interest Rate Models with Mean Reversion

Model	Formula
Vasicek	dr = b * (a – r) dt + s dZ
Hull-White	dr = b * (a(t) / b – r) dt + s dZ
Cox-Ingersoll-Ross	dr = b * (a – r) dt + s * sqrt(r) dZ
Brennan & Schwartz	dr = b * (a – r) dt + s * r dZ
Longstaff	dr = b * (a – sqrt(r)) dt + s * sqrt(r) dZ

Mean reversion is a common way of constraining time series models to be stationary, while still allowing a close relationship between consecutive values.

But there are problems with the use of such models for interest rates. This paper uses monthly time series data from Australia and the United States of America to illustrate the shortfalls of standard time series models to describe interest rates, and suggests a solution to these problems via regime switching models.

Unit Roots

Statistical tests of the null hypothesis that a time series is non-stationary against the alternative that it is stationary are called "unit root" tests. The term "unit root" derives from the fact that an ARMA process is non-stationary if the characteristic polynomial has a root that does not lie within the unit circle of complex numbers.

A unit root can occur in an AR(1) model when the reversion factor equals one. In such cases the time series is not stationary and is more of a random walk than a mean reversionary series.

Sherris, Tedesco and Zehnwirth (1997) state that Australian interest rates may have a unit root.

There is a choice of statistical tests for unit roots, all of which have the existence of a unit root as the null hypothesis that the test then seeks to disprove this hypothesis. In this paper I have chosen to use the Dickey-Fuller (Case 2: Constant Term but No Time Trend Included in the Regression; True Process is a Random Walk) statistical test as recommended by Hamilton (1994). In this test one expresses the time series in the form:

Figure 1: Alternate AR(1) Formula

$Y_t = \alpha + \beta * Y_{t-1} + e_t$

Then one tests whether β is significant using a test value that is a function of the least squares estimator of β and the number of observations.

Figure 2: Dickey-Fuller Test Statistic

$T = (n - 1) * \beta$
where n is the number of observations and T is the sample test statistic

The critical values for this test statistic were tabulated by Dickey (1976).

Table 2: Test Results for Unit Roots

Series	Sample Test Statistic	Critical Value	Conclusion
Aust 1959 onwards	-7.21	-16.80	May have unit root
Aust 1992 onwards	-0.96	-16.80	May have unit root
USA 1970 onwards	-4.23	-16.80	May have unit root
USA 1990 onwards	-3.07	-16.80	May have unit root

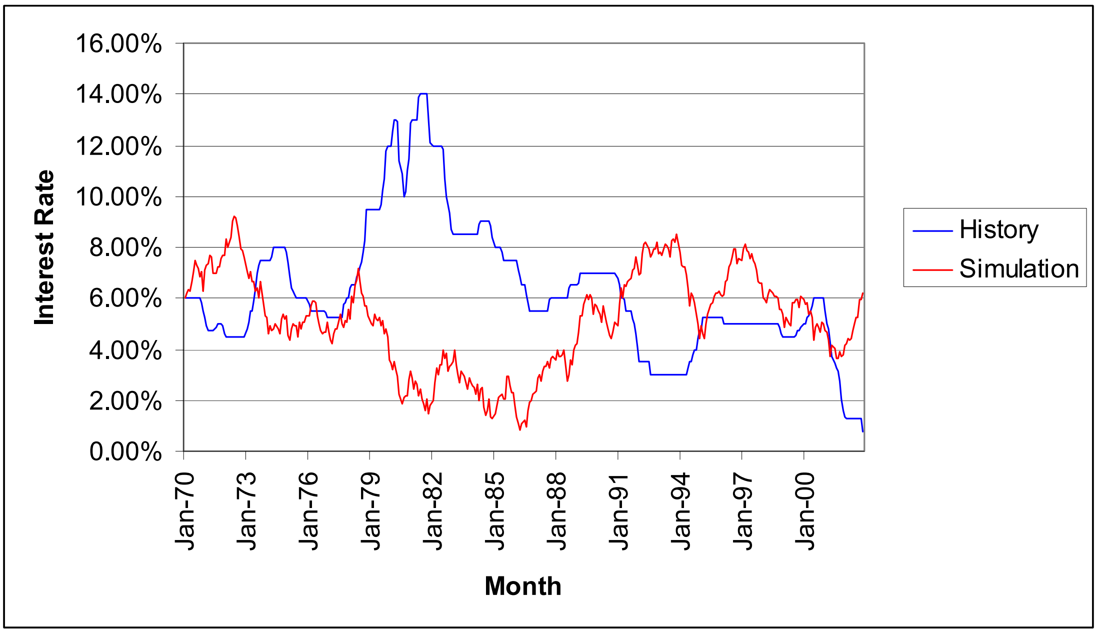
In each of the tests I was unable to reject the hypothesis that the series has a unit root and in each case the sample autocorrelation estimate exceeded 98%. With such a high autocorrelation coefficient, even if the series does not have a unit root, the mean reversionary effect would be minimal.

Since the null hypothesis was that a unit root existed, these tests do not prove the existence of a unit root, and do not disprove the assumption of an autoregressive time series. However, they highlight the danger of applying an autoregressive model to this data.

Movements in Interest Rates

In order to reject autoregressive models for interest rates in Australia and USA, one must use a statistical test that has an autoregressive process as the null hypothesis, and which is able to disprove that hypothesis.

Graph 1: USA Data vs. AR(1) Simulation



The form of this test was suggested by a graphical presentation of the time series. The US and Australian data appears to be partially cyclic, whereas a standard AR(1) process will not appear cyclic. The reason that the interest rate series appear cyclic may be because the changes in interest rates from month to month may be correlated. We can test the sample correlation coefficient of the changes in interest rates against the probability that such a sample value would occur if the time series was AR(1) with best fit parameters. I have used Monte Carlo simulation to estimate the probabilities of the sample correlation of interest rates being as different from the mean of the changes in rates if they followed an AR(1) process.

Table 3: Test Results for Delta Interest Rates

Series	**Corr of Δ Interest**	**Probability**	**Conclusion**
Aust 1959 onwards	-10%	0.018	Not AR(1)
Aust 1992 onwards	42%	0.000	Not AR(1)
USA 1970 onwards	52%	0.000	Not AR(1)
USA 1990 onwards	56%	0.000	Not AR(1)

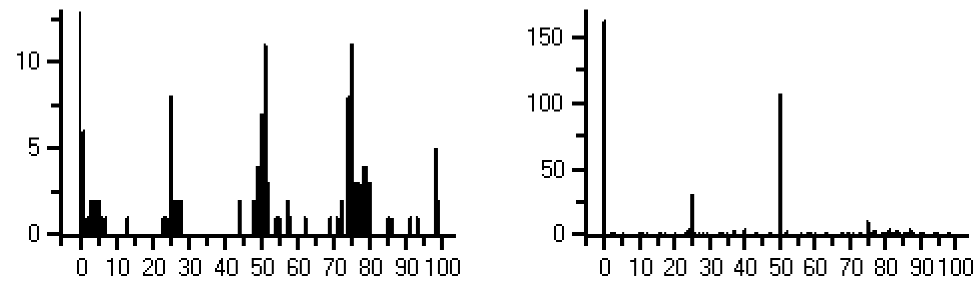
In each time series we are able to reject the null hypothesis that the series followed an AR(1) process. Changes in interest rate from month to month are strongly correlated with one another. One may ask why the Australian data changes from a negative to a positive correlation, while the USA data remains strongly positive. The answer lies in a change in interest rate regulation by the Reserve Bank in Australia during the early 1990s whereby the official overnight cash rate became a gazetted rate rather than a market rate. In the USA the rate was gazetted during the entire period being reviewed.

Continuous vs. Discrete Models

Since the cash rate is a gazetted rate in both USA and Australia, cash rates will not change over periods of time shorter than the central banks' regular policy reviews for these rates. Since I am using a monthly time series, and the central banks typically meet monthly to review cash rate policy, this should not pose too much of a problem.

But the effects of gazetting are felt beyond the time period between monthly policy reviews. In a very liquid market in which rates are set by the market, snapshots of market rates should give rates that are spread fairly evenly over different decimal values. Instead, we see rates that clump around multiples of 0.25%.

Figure 3: Clumping of Interest Rates (Aust and USA Respectively)



In both of these graphs the x axis represents the first two decimal places of the interest rate, while the y axis represents the number of observations. In Australia there is a clear preference for rates that are a multiple of 0.25% with a slight bias towards rates that are a multiple of 1.00%. The variations from the exact numbers are due to the way that the data was compiled – it is the average rate over the entire month, whereas an end of month rate would show even more clumping. In the USA the bias towards rates that are a multiple of 0.25% is much clearer. There is also an indication of a bias towards rates that are multiples of 0.50% and 1.00%.

We can do a formal statistical test to confirm this, doing a chi squared test of actual vs. expected observations against a null hypothesis that rates do not clump around any particular decimal value.

Table 4: Statistical Tests for Clumping

Series	Sample Statistic	Critical Value	Conclusion
Aust 1959 onwards	188.3	123.2	Rates are clumped
Aust 1992 onwards	474.3	123.2	Rates are clumped
USA 1970 onwards	9410.3	123.2	Rates are clumped
USA 1990 onwards	4537.3	123.2	Rates are clumped

In each case the null hypothesis is rejected. This also invalidates autoregressive models, because those models are continuous.

Stagnant Interest Rates

Another feature of interest rates is that sometimes rates do not move for many months at a time. One could deal with clumping by rounding an autoregressive time series to the nearest multiple of 0.25%, but the rates would still change quite frequently.

I have developed a statistical test for rate stagnancy. Assume a state space model in which the underlying state time series is an AR(1) with best fit parameters, and the observed series is the underlying time series rounded to the nearest multiple of 0.25%. One can then use this assumption as the null hypothesis, and then compare the length of the longest period of stagnancy in rates with the probability of such a length under the null hypothesis. I have used Monte Carlo methods to estimate this probability.

Table 5: Statistical Tests for Stagnancy

Series	Longest Run	Probability	Conclusion
Aust 1959 onwards	18	0.000	Not AR(1)
Aust 1992 onwards	18	0.000	Not AR(1)
USA 1970 onwards	32	0.000	Not AR(1)
USA 1990 onwards	32	0.000	Not AR(1)

In each data series there are lengths of time exceeding a year in which interest rates do not change. In the USA the longest period of interest rate stagnancy is almost three years long.

Conclusion: Autoregressive Models Are Not Appropriate for Interest Rates

The interest rate series have four characteristics that preclude the use of autoregressive models:

1. the possible existence of unit roots
2. statistically significant autocorrelation of changes in rates
3. statistically significant clumping of rates – more like a discrete series
4. statistically significant periods of stagnancy

An Alternative: Regime Switching Models

For the purposes of this paper, a regime is a time period in which a particular set of rules apply. Once the regime is over those rules no longer apply, and another set of rules, belonging to another regime, come into force.

A real life example of regimes is a tennis match. There are four distinct regimes:

1. Player A is at the south end of the court and is serving

2. Player A is at the south end of the court and is receiving
3. Player A is at the north end of the court and is serving
4. Player A is at the north end of the court and is receiving

The rules for each regime vary, changing which direction in which Player A must hit the ball (either north or south) and whether Player A begins the play (whether Player A serves).

Regime switching models are characterised by a number of distinct and discrete regimes within which different model parameters apply. From time to time the model switches from one regime to another and the characteristics of the observations change to match the underlying characteristics of that particular regime. These regime switches represent structural changes occurring in the process being modelled.

The use of regime switching models began in the field of economics. Kim and Nelson (1996) describe the development of regime switching models within econometrics. Of particular interest is the use of regime switching models to describe economic cycles in which economic growth and recession are the two possible regimes. An economy can be seen as cycling between boom and bust. A regime switching model allows the econometrician to measure the different characteristics of growth and recession.

Regime switching models are not unknown to actuaries. Harris (1999) describes an actuarial application of regime switching models in investment modelling. He derives a regime switching vector autoregression model that describes investment returns for different classes of assets. His model has different regimes for stability and uncertainty. Priest (2001) developed a saw tooth regime switching model to describe both building construction activity and also insurance cycles. Ang and Bekaert (2002) have published three papers in recent months, two of which deal with regime switches for interest rate series.

The reason that regime switching models have only come into use in relatively recent times is that it is difficult to fit parameters for regime switching models using traditional least squares and maximum likelihood methods. At no point of time can one directly observe which regime a process lies in. Regime choices can only be inferred by observed data. So the regime values must be treated as unobserved data.

The Alternate Model

I propose a regime switching model to deal with four characters of interest rate series:

1. the possible existence of unit roots
2. statistically significant autocorrelation of changes in rates
3. statistically significant clumping of rates – more like a discrete series
4. statistically significant periods of stagnancy

Firstly, I propose that the short rate follows a discrete process, that interest rates will always be a multiple of 0.25%. Interest rates can then jump from one multiple of 0.25% to another.

Secondly, I propose that interest rates tend to move towards an attractor mean value, but that the attractor mean value will occasionally jump to a different value. The value of the attractor mean is the regime in the regime switching model. I propose that the probability of a change in attractor mean is constant over time.

Thirdly, I propose that the probability of a particular change in interest rates from month to month follows a truncated Poisson distribution for the number of multiples of 0.25% that the interest rate will move in that month. The reason that the number of multiples is truncated is because interest rates will not overshoot their attractor mean. So if an interest rate is already at its attractor mean, then it has a probability of 1 of staying at the same rate.

Finally I propose that different attractor mean values are sampled from a log normal distribution, but with the values rounded to the nearest 0.25%. This ensures that high interest regimes occur less often than low interest regimes.

Parameter Estimation for Regime Switching Models

As soon as we bring regime switches into a model, the model becomes non-linear, which adds considerable complexity of estimating the parameters. Kim and Nelson (1999) describe the use of the Gibbs sampling algorithm for determining the parameters of regime switching models.

Gibbs sampling is a Markov Chain Monte Carlo algorithm for approximating joint statistical distributions by sampling from conditional statistical distributions. It uses a Bayesian approach – so it uses probabilities that are conditional upon some values being certain or fixed, and combines these probabilities so that all of the values are variable.

Suppose you want to estimate a multivariate probability function $f(x_1, x_2, \ldots, x_N)$ but that doing so directly is computationally unfeasible. The Gibbs sampler algorithm can produce samples from this multivariate distribution using the following algorithm:

1. Start with an arbitrary set of starting values $(z_1, z_2, \ldots, z_N)$
2. Draw a sample from $f(z_1 \mid z_2, \ldots, z_N)$
3. Draw a sample from $f(z_2 \mid z_1, z_3, \ldots, z_N)$
4. Iterate through j (where steps 2 & 3 are the first two iterations) to sample from $f(z_j \mid z_1, z_2, \ldots, z_{j-1}, z_{j+1}, \ldots, z_N)$
5. The last iteration of j is $f(z_N \mid z_1, z_2, \ldots, z_{N-1})$

Steps 2 to 5 can be repeated a number of times. The distribution of the set of sample values stored at step 5 will converge to $f(x_1, x_2, \ldots, x_N)$.

You can store the sample values and use them to approximate $f(x_1, x_2, \ldots, x_N)$. But you wouldn't store the early sample values because it takes a while for the sampling series to converge to the target distribution when the starting values are arbitrary. In my experience, the starting values cannot be totally arbitrary because it can prevent the series from converging at all.

Sampling the Probability of a Regime Switch

The probability of a regime switch in a particular period must lie in the range 0 to 1 because it is a probability. For my prior distribution I have used an uninformative rectangular distribution, whereby all values from 0 to 1 are equally likely.

Then, (using Gibbs sampling) assume that we know exactly the location of each regime switch (even though these are unobserved variables). The posterior distribution of the regime switch probability becomes a Beta distribution.

Figure 4: Posterior Distribution of Regime Switch Probability

$L(p \mid S_T) = p^n (1 - p)^n$ which is a Beta (1 + n, 1 + n) distribution.

Sampling the Poisson Rate Change Parameter

The Poisson λ parameter must be a value greater than 0. I have assumed no prior distribution other than the constraint that λ must be greater than zero.

Then, (using Gibbs sampling) assume that we know the attractor mean at each point of time (even though these are unobserved variables). The posterior distribution of the λ parameter can be found using the likelihood function for a histogram of the number of multiples of 0.25% that a rate change has. For practical purposes, we exclude those months where the interest rate already equals the attractor mean.

Figure 5: Posterior Distribution of Poisson Rate Change Parameter

$L(\lambda \mid S_T) = \Pi_n [\lambda^n \exp(-\lambda) / n!]^{Nn}$
Where n is the number of multiples of 0.25% and Nn is the number of observations with that number of multiples of 0.25%

While the likelihood formula implies an infinite series, the series factor simplifies to 1.0 whenever Nn = 0 i.e. for counts where there are no observations. So we only need to multiply the series for the histogram bins that have an observation within it.

I have used a numerical technique to take sample values from the distribution defined by this likelihood.

Sampling the Attractor Mean Location

The attractor mean location must be a positive value that is a multiple of 0.25%. In order to make the arithmetic tractable, I have also constrained the attractor mean to lie within the range of observed interest rates. Initially my prior distribution ignored the discretised log-normal assumption, and simply assigned equal probabilities to each interest rate in this range.

Then, bringing in the regime switch locations, we can further constrain the range of possible values. If the regime moves downward, then the attractor mean cannot lie above the minimum observed rate during that time. If the regime moves upward, then the attractor mean cannot lie below the maximum observed rate during that time.

Finally, (using Gibbs sampling) we assume that we know the Poisson parameter for rate switches. The likelihood of different attractor means is then the multiplicative series of the Poisson probabilities.

Figure 6: Posterior Distribution for Attractor Means

$L(a \mid S_T) = \Pi_m [\lambda^n \exp(-\lambda) / n!]$
Where n is the number of multiples of 0.25% that the rate changed in the month and the series extends over the range of months that the regime spans.

Sampling the Regime Switch Location

I was unable to determine a robust way of allowing the regime switch locations to be random variables. So they have been kept constant during the Gibbs sampling process, using values that are implied from the historical data.

Model Results

I have not chosen to fit parameters to Australian data prior to 1992 because cash rates were determined on a market basis prior to that period. If I were to fit a model to that period then I would require the model to change when cash rates became gazetted.

Table 6: Model Parameters for Australia from 1992

Parameter	Mean	Standard Deviation
Poisson parameter for rate changes	0.545	0.087
Probability of regime switch	0.040	0.018
Attractor mean values	5.61%	1.40%
Attractor means autocorrelation	-0.67	0.16

There is a probability of 0.15 that the absolute value of the sample autocorrelation of attractor means will reach 0.67 assuming that the underlying values are not actually correlated over time. This means that we cannot reject the hypothesis that the attractor means are not autoregressive.

Table 7: Model Parameters for USA from 1970

Parameter	Mean	Standard Deviation
Poisson parameter for rate changes	0.645	0.049
Probability of regime switch	0.043	0.010
Attractor mean values	7.01%	4.09%
Attractor means autocorrelation	-0.32	0.20

There is a probability of 0.27 that the absolute value of the sample autocorrelation of attractor means will reach 0.32 assuming that the underlying values are not actually correlated over time. This means that we cannot reject the hypothesis that the attractor means are not autoregressive.

Table 8: Model Parameters for USA from 1990

Parameter	Mean	Standard Deviation
Poisson parameter for rate changes	0.602	0.083
Probability of regime switch	0.032	0.014
Attractor mean values	3.98%	2.02%
Attractor means autocorrelation	-0.73	0.08

There is a probability of 0.10 that the absolute value of the sample autocorrelation of attractor means will reach 0.73 assuming that the underlying values are not actually correlated over time. This means that we cannot reject the hypothesis that the attractor means are not autoregressive.

The autocorrelation of attractor means was not sufficiently statistically significant in any of the cases to reject a null hypothesis that they possess no autoregressive behaviour.

It is noteworthy that the regime switching and rate changing parameters are fairly consistent across time and between Australia and USA.

Further Research

The suggested model is by no means complete. As mentioned earlier in this paper, I was unable to find a way of allowing the location of the regime switches to move, without losing convergence of the results. But there are other aspects of interest rates that could be incorporated into this model:

- Heteroskedasticity between low rates and high rates
- Yield curves

Conclusion

Autogressive time series models are not appropriate for modelling interest rate series. I have applied two well known statistical tests to show the possible existence of unit roots, and the existence of clumping of rates around particular decimal values. Then I have developed two statistical tests, one for autocorrelation of changes in values, and the other for stagnancy of values. These tests both reject the hypothesis that rates follow an AR(1) process.

As an alternative for autoregressive time series models, I have suggested a regime switching model where by discrete spot rates tend to move towards an attractor mean that changes from regime to regime. This model can be difficult to fit, but has some attractive properties. Furthermore, once one fits parameters to this model, it can be shown that not only are interest rates not mean reversionary, but that the attractor means may also lack mean reversionary properties.

Bibliography

Ang, A., Bekaert, G. “Regime Switches in Interest Rates”, *Journal of Business and Economic Statistics*, 20, 2, (2002) pp163-182

Ang, A. Bekaert, G. “Short Rate Nonlinearities and Regime Switches”, *Journal of Economic Dynamics and Control*, 26, 7-8, (2002) pp1243-1274

Ang, A. Bekaert, G. “International Asset Allocation with Regime Shifts”, *Review of Financial Studies*, 15, 4, (2002) pp 1137-1187

Dickey, D., Fuller, W. “Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root.”, *Econometrica* 49 (1981) p1063

Fuller, A. “Introduction to Statistical Time Series”, Wiley, New York (1976) p371

Hamilton, J. “Time Series Analysis” Princeton University Press, New Jersey (1994)

Kim C.J. and Nelson C.R. (1999) “State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications”. The MIT Press, Cambridge, Massachusetts

Priest, C. “Regime Switching and Cycles”, IAA General Insurance Seminar 2001

Sherris, M., Tedesco, L. and Zehnwirth, B. "Stochastic Investment Models: Unit Roots, Cointegration, State Space and GARCH Models.” *Actuarial Research Clearing House* no. 1, (1997): 95-144.

Appendix A: Australian Interest Rate History

Jun-59	3.11%	Jun-64	3.71%	Jun-69	4.59%	Jun-74	6.82%
Jul-59	3.10%	Jul-64	3.77%	Jul-69	4.58%	Jul-74	8.15%
Aug-59	3.06%	Aug-64	3.74%	Aug-69	4.40%	Aug-74	9.47%
Sep-59	3.00%	Sep-64	3.74%	Sep-69	4.54%	Sep-74	7.15%
Oct-59	2.96%	Oct-64	3.83%	Oct-69	4.65%	Oct-74	8.01%
Nov-59	2.93%	Nov-64	3.69%	Nov-69	4.33%	Nov-74	7.97%
Dec-59	2.94%	Dec-64	3.46%	Dec-69	4.40%	Dec-74	7.00%
Jan-60	2.89%	Jan-65	3.53%	Jan-70	4.63%	Jan-75	8.54%
Feb-60	2.93%	Feb-65	3.47%	Feb-70	4.44%	Feb-75	7.85%
Mar-60	2.99%	Mar-65	3.63%	Mar-70	4.45%	Mar-75	7.67%
Apr-60	3.13%	Apr-65	4.31%	Apr-70	5.48%	Apr-75	6.09%
May-60	3.20%	May-65	4.36%	May-70	5.44%	May-75	6.96%
Jun-60	3.25%	Jun-65	4.16%	Jun-70	6.12%	Jun-75	6.72%
Jul-60	3.38%	Jul-65	4.31%	Jul-70	5.72%	Jul-75	6.85%
Aug-60	3.52%	Aug-65	4.30%	Aug-70	5.46%	Aug-75	7.08%
Sep-60	3.74%	Sep-65	4.35%	Sep-70	5.18%	Sep-75	7.52%
Oct-60	3.58%	Oct-65	4.06%	Oct-70	5.54%	Oct-75	6.35%
Nov-60	3.68%	Nov-65	3.96%	Nov-70	4.97%	Nov-75	6.64%
Dec-60	3.94%	Dec-65	4.07%	Dec-70	4.90%	Dec-75	7.23%
Jan-61	3.73%	Jan-66	3.97%	Jan-71	5.29%	Jan-76	6.80%
Feb-61	3.43%	Feb-66	3.97%	Feb-71	4.94%	Feb-76	7.17%
Mar-61	3.82%	Mar-66	4.49%	Mar-71	4.97%	Mar-76	6.70%
Apr-61	4.23%	Apr-66	4.26%	Apr-71	5.67%	Apr-76	7.09%
May-61	4.19%	May-66	4.14%	May-71	5.58%	May-76	7.71%
Jun-61	4.17%	Jun-66	4.73%	Jun-71	5.91%	Jun-76	8.02%
Jul-61	3.76%	Jul-66	4.48%	Jul-71	5.87%	Jul-76	7.57%
Aug-61	3.45%	Aug-66	4.27%	Aug-71	5.57%	Aug-76	7.75%
Sep-61	3.25%	Sep-66	4.44%	Sep-71	5.71%	Sep-76	7.46%
Oct-61	3.36%	Oct-66	4.32%	Oct-71	5.74%	Oct-76	7.28%
Nov-61	3.62%	Nov-66	4.14%	Nov-71	5.11%	Nov-76	7.18%
Dec-61	3.50%	Dec-66	4.53%	Dec-71	5.17%	Dec-76	6.37%
Jan-62	3.13%	Jan-67	3.92%	Jan-72	5.33%	Jan-77	7.46%
Feb-62	3.43%	Feb-67	4.25%	Feb-72	4.87%	Feb-77	7.42%
Mar-62	3.46%	Mar-67	4.12%	Mar-72	5.04%	Mar-77	7.58%
Apr-62	3.64%	Apr-67	4.21%	Apr-72	4.78%	Apr-77	7.36%
May-62	3.62%	May-67	4.23%	May-72	5.18%	May-77	8.64%
Jun-62	3.45%	Jun-67	4.46%	Jun-72	5.18%	Jun-77	9.52%
Jul-62	3.34%	Jul-67	4.26%	Jul-72	4.44%	Jul-77	9.34%
Aug-62	3.59%	Aug-67	4.17%	Aug-72	4.41%	Aug-77	9.67%
Sep-62	3.70%	Sep-67	4.22%	Sep-72	4.24%	Sep-77	9.47%
Oct-62	3.49%	Oct-67	4.08%	Oct-72	3.90%	Oct-77	8.85%
Nov-62	3.50%	Nov-67	3.66%	Nov-72	4.29%	Nov-77	9.16%
Dec-62	3.69%	Dec-67	4.16%	Dec-72	4.12%	Dec-77	9.15%
Jan-63	3.23%	Jan-68	3.88%	Jan-73	3.86%	Jan-78	8.75%
Feb-63	3.57%	Feb-68	4.31%	Feb-73	4.21%	Feb-78	8.42%
Mar-63	3.50%	Mar-68	4.10%	Mar-73	4.38%	Mar-78	7.95%
Apr-63	3.59%	Apr-68	4.24%	Apr-73	4.14%	Apr-78	7.67%
May-63	3.54%	May-68	4.23%	May-73	4.16%	May-78	9.11%
Jun-63	3.75%	Jun-68	4.29%	Jun-73	4.70%	Jun-78	9.04%
Jul-63	3.31%	Jul-68	4.19%	Jul-73	4.60%	Jul-78	8.81%
Aug-63	3.31%	Aug-68	4.16%	Aug-73	5.11%	Aug-78	9.01%
Sep-63	3.38%	Sep-68	4.07%	Sep-73	5.75%	Sep-78	9.18%
Oct-63	3.28%	Oct-68	4.18%	Oct-73	6.90%	Oct-78	9.40%
Nov-63	3.13%	Nov-68	3.83%	Nov-73	5.79%	Nov-78	7.97%
Dec-63	3.08%	Dec-68	4.18%	Dec-73	6.20%	Dec-78	8.31%
Jan-64	3.02%	Jan-69	3.94%	Jan-74	6.48%	Jan-79	8.18%
Feb-64	3.18%	Feb-69	4.07%	Feb-74	6.82%	Feb-79	7.83%
Mar-64	3.38%	Mar-69	4.13%	Mar-74	7.60%	Mar-79	8.18%
Apr-64	3.55%	Apr-69	4.48%	Apr-74	8.16%	Apr-79	8.61%

Jun-79	8.71%	Jun-84	12.33%	Jun-89	17.73%	Jun-94	4.77%
Jul-79	10.06%	Jul-84	12.19%	Jul-89	17.92%	Jul-94	4.74%
Aug-79	10.63%	Aug-84	11.61%	Aug-89	17.86%	Aug-94	5.13%
Sep-79	9.41%	Sep-84	11.07%	Sep-89	18.06%	Sep-94	5.50%
Oct-79	9.05%	Oct-84	11.14%	Oct-89	18.05%	Oct-94	5.79%
Nov-79	8.99%	Nov-84	11.52%	Nov-89	18.18%	Nov-94	6.51%
Dec-79	9.50%	Dec-84	12.01%	Dec-89	18.16%	Dec-94	7.03%
Jan-80	9.24%	Jan-85	11.27%	Jan-90	17.81%	Jan-95	7.49%
Feb-80	8.86%	Feb-85	11.64%	Feb-90	16.80%	Feb-95	7.48%
Mar-80	10.40%	Mar-85	13.97%	Mar-90	16.43%	Mar-95	7.49%
Apr-80	10.68%	Apr-85	15.39%	Apr-90	15.17%	Apr-95	7.51%
May-80	13.72%	May-85	15.84%	May-90	15.02%	May-95	7.50%
Jun-80	13.73%	Jun-85	18.92%	Jun-90	15.05%	Jun-95	7.51%
Jul-80	12.86%	Jul-85	15.48%	Jul-90	15.02%	Jul-95	7.51%
Aug-80	12.42%	Aug-85	16.50%	Aug-90	14.07%	Aug-95	7.51%
Sep-80	11.23%	Sep-85	16.41%	Sep-90	14.05%	Sep-95	7.49%
Oct-80	9.83%	Oct-85	16.09%	Oct-90	13.47%	Oct-95	7.48%
Nov-80	10.33%	Nov-85	18.12%	Nov-90	13.05%	Nov-95	7.50%
Dec-80	10.88%	Dec-85	19.39%	Dec-90	12.68%	Dec-95	7.51%
Jan-81	10.68%	Jan-86	18.90%	Jan-91	12.02%	Jan-96	7.50%
Feb-81	11.00%	Feb-86	18.58%	Feb-91	12.01%	Feb-96	7.50%
Mar-81	11.99%	Mar-86	17.29%	Mar-91	12.02%	Mar-96	7.52%
Apr-81	13.09%	Apr-86	17.15%	Apr-91	11.58%	Apr-96	7.49%
May-81	14.19%	May-86	15.11%	May-91	10.99%	May-96	7.51%
Jun-81	15.33%	Jun-86	15.43%	Jun-91	10.54%	Jun-96	7.51%
Jul-81	15.44%	Jul-86	14.71%	Jul-91	10.48%	Jul-96	7.51%
Aug-81	15.00%	Aug-86	17.78%	Aug-91	10.52%	Aug-96	7.01%
Sep-81	14.68%	Sep-86	17.70%	Sep-91	9.59%	Sep-96	7.01%
Oct-81	14.34%	Oct-86	16.64%	Oct-91	9.49%	Oct-96	7.00%
Nov-81	14.54%	Nov-86	16.35%	Nov-91	8.64%	Nov-96	6.58%
Dec-81	15.62%	Dec-86	15.50%	Dec-91	8.51%	Dec-96	6.23%
Jan-82	14.99%	Jan-87	16.62%	Jan-92	7.75%	Jan-97	6.04%
Feb-82	15.20%	Feb-87	16.43%	Feb-92	7.52%	Feb-97	6.01%
Mar-82	16.08%	Mar-87	16.23%	Mar-92	7.54%	Mar-97	6.04%
Apr-82	19.09%	Apr-87	14.80%	Apr-92	7.51%	Apr-97	6.05%
May-82	18.39%	May-87	14.26%	May-92	6.69%	May-97	5.91%
Jun-82	17.58%	Jun-87	13.18%	Jun-92	6.57%	Jun-97	5.57%
Jul-82	16.51%	Jul-87	12.43%	Jul-92	5.93%	Jul-97	5.44%
Aug-82	20.77%	Aug-87	12.16%	Aug-92	5.86%	Aug-97	4.98%
Sep-82	16.10%	Sep-87	11.80%	Sep-92	5.74%	Sep-97	4.98%
Oct-82	15.70%	Oct-87	11.30%	Oct-92	5.74%	Oct-97	4.98%
Nov-82	14.00%	Nov-87	11.69%	Nov-92	5.77%	Nov-97	5.00%
Dec-82	11.46%	Dec-87	11.30%	Dec-92	5.79%	Dec-97	5.03%
Jan-83	12.36%	Jan-88	10.62%	Jan-93	5.75%	Jan-98	5.00%
Feb-83	12.68%	Feb-88	10.66%	Feb-93	5.74%	Feb-98	4.98%
Mar-83	16.73%	Mar-88	10.87%	Mar-93	5.62%	Mar-98	5.00%
Apr-83	12.61%	Apr-88	11.30%	Apr-93	5.25%	Apr-98	4.98%
May-83	11.90%	May-88	12.58%	May-93	5.26%	May-98	5.00%
Jun-83	11.74%	Jun-88	13.07%	Jun-93	5.27%	Jun-98	5.07%
Jul-83	10.35%	Jul-88	12.71%	Jul-93	5.24%	Jul-98	5.01%
Aug-83	10.67%	Aug-88	12.91%	Aug-93	4.77%	Aug-98	5.00%
Sep-83	10.98%	Sep-88	13.07%	Sep-93	4.72%	Sep-98	4.99%
Oct-83	10.24%	Oct-88	13.41%	Oct-93	4.71%	Oct-98	4.99%
Nov-83	9.97%	Nov-88	14.33%	Nov-93	4.75%	Nov-98	5.00%
Dec-83	9.76%	Dec-88	14.59%	Dec-93	4.79%	Dec-98	4.80%
Jan-84	9.13%	Jan-89	14.82%	Jan-94	4.74%	Jan-99	4.75%
Feb-84	9.78%	Feb-89	15.77%	Feb-94	4.74%	Feb-99	4.74%
Mar-84	12.55%	Mar-89	16.51%	Mar-94	4.78%	Mar-99	4.75%
Apr-84	15.15%	Apr-89	16.71%	Apr-94	4.76%	Apr-99	4.75%

Jun-99	4.80%
Jul-99	4.76%
Aug-99	4.76%
Sep-99	4.78%
Oct-99	4.79%
Nov-99	5.05%
Dec-99	5.01%
Jan-00	5.01%
Feb-00	5.51%
Mar-00	5.50%
Apr-00	5.78%
May-00	6.02%
Jun-00	6.00%
Jul-00	6.00%
Aug-00	6.25%
Sep-00	6.28%
Oct-00	6.27%
Nov-00	6.26%
Dec-00	6.25%
Jan-01	6.25%
Feb-01	5.85%
Mar-01	5.55%
Apr-01	5.06%
May-01	5.00%
Jun-01	5.00%
Jul-01	5.00%
Aug-01	5.00%
Sep-01	4.78%
Oct-01	4.52%
Nov-01	4.50%
Dec-01	4.28%
Jan-02	4.25%
Feb-02	4.25%
Mar-02	4.25%
Apr-02	4.25%
May-02	4.44%
Jun-02	4.72%
Jul-02	4.75%
Aug-02	4.75%
Sep-02	4.75%
Oct-02	4.75%
Nov-02	4.75%

Appendix B: USA Interest Rate History

Jan-70	6.00%	Jan-75	7.40%	Jan-80	12.00%	Jan-85	8.00%
Feb-70	6.00%	Feb-75	6.82%	Feb-80	12.52%	Feb-85	8.00%
Mar-70	6.00%	Mar-75	6.40%	Mar-80	13.00%	Mar-85	8.00%
Apr-70	6.00%	Apr-75	6.25%	Apr-80	13.00%	Apr-85	8.00%
May-70	6.00%	May-75	6.12%	May-80	12.94%	May-85	7.81%
Jun-70	6.00%	Jun-75	6.00%	Jun-80	11.40%	Jun-85	7.50%
Jul-70	6.00%	Jul-75	6.00%	Jul-80	10.87%	Jul-85	7.50%
Aug-70	6.00%	Aug-75	6.00%	Aug-80	10.00%	Aug-85	7.50%
Sep-70	6.00%	Sep-75	6.00%	Sep-80	10.17%	Sep-85	7.50%
Oct-70	6.00%	Oct-75	6.00%	Oct-80	11.00%	Oct-85	7.50%
Nov-70	5.85%	Nov-75	6.00%	Nov-80	11.47%	Nov-85	7.50%
Dec-70	5.52%	Dec-75	6.00%	Dec-80	12.87%	Dec-85	7.50%
Jan-71	5.23%	Jan-76	5.79%	Jan-81	13.00%	Jan-86	7.50%
Feb-71	4.91%	Feb-76	5.50%	Feb-81	13.00%	Feb-86	7.50%
Mar-71	4.75%	Mar-76	5.50%	Mar-81	13.00%	Mar-86	7.10%
Apr-71	4.75%	Apr-76	5.50%	Apr-81	13.00%	Apr-86	6.83%
May-71	4.75%	May-76	5.50%	May-81	13.87%	May-86	6.50%
Jun-71	4.75%	Jun-76	5.50%	Jun-81	14.00%	Jun-86	6.50%
Jul-71	4.88%	Jul-76	5.50%	Jul-81	14.00%	Jul-86	6.16%
Aug-71	5.00%	Aug-76	5.50%	Aug-81	14.00%	Aug-86	5.82%
Sep-71	5.00%	Sep-76	5.50%	Sep-81	14.00%	Sep-86	5.50%
Oct-71	5.00%	Oct-76	5.50%	Oct-81	14.00%	Oct-86	5.50%
Nov-71	4.90%	Nov-76	5.43%	Nov-81	13.03%	Nov-86	5.50%
Dec-71	4.63%	Dec-76	5.25%	Dec-81	12.10%	Dec-86	5.50%
Jan-72	4.50%	Jan-77	5.25%	Jan-82	12.00%	Jan-87	5.50%
Feb-72	4.50%	Feb-77	5.25%	Feb-82	12.00%	Feb-87	5.50%
Mar-72	4.50%	Mar-77	5.25%	Mar-82	12.00%	Mar-87	5.50%
Apr-72	4.50%	Apr-77	5.25%	Apr-82	12.00%	Apr-87	5.50%
May-72	4.50%	May-77	5.25%	May-82	12.00%	May-87	5.50%
Jun-72	4.50%	Jun-77	5.25%	Jun-82	12.00%	Jun-87	5.50%
Jul-72	4.50%	Jul-77	5.25%	Jul-82	11.81%	Jul-87	5.50%
Aug-72	4.50%	Aug-77	5.27%	Aug-82	10.68%	Aug-87	5.50%
Sep-72	4.50%	Sep-77	5.75%	Sep-82	10.00%	Sep-87	5.95%
Oct-72	4.50%	Oct-77	5.80%	Oct-82	9.68%	Oct-87	6.00%
Nov-72	4.50%	Nov-77	6.00%	Nov-82	9.35%	Nov-87	6.00%
Dec-72	4.50%	Dec-77	6.00%	Dec-82	8.73%	Dec-87	6.00%
Jan-73	4.77%	Jan-78	6.37%	Jan-83	8.50%	Jan-88	6.00%
Feb-73	5.05%	Feb-78	6.50%	Feb-83	8.50%	Feb-88	6.00%
Mar-73	5.50%	Mar-78	6.50%	Mar-83	8.50%	Mar-88	6.00%
Apr-73	5.50%	Apr-78	6.50%	Apr-83	8.50%	Apr-88	6.00%
May-73	5.90%	May-78	6.84%	May-83	8.50%	May-88	6.00%
Jun-73	6.33%	Jun-78	7.00%	Jun-83	8.50%	Jun-88	6.00%
Jul-73	6.98%	Jul-78	7.23%	Jul-83	8.50%	Jul-88	6.00%
Aug-73	7.29%	Aug-78	7.43%	Aug-83	8.50%	Aug-88	6.37%
Sep-73	7.50%	Sep-78	7.83%	Sep-83	8.50%	Sep-88	6.50%
Oct-73	7.50%	Oct-78	8.26%	Oct-83	8.50%	Oct-88	6.50%
Nov-73	7.50%	Nov-78	9.50%	Nov-83	8.50%	Nov-88	6.50%
Dec-73	7.50%	Dec-78	9.50%	Dec-83	8.50%	Dec-88	6.50%
Jan-74	7.50%	Jan-79	9.50%	Jan-84	8.50%	Jan-89	6.50%
Feb-74	7.50%	Feb-79	9.50%	Feb-84	8.50%	Feb-89	6.59%
Mar-74	7.50%	Mar-79	9.50%	Mar-84	8.50%	Mar-89	7.00%
Apr-74	7.60%	Apr-79	9.50%	Apr-84	8.87%	Apr-89	7.00%
May-74	8.00%	May-79	9.50%	May-84	9.00%	May-89	7.00%
Jun-74	8.00%	Jun-79	9.50%	Jun-84	9.00%	Jun-89	7.00%
Jul-74	8.00%	Jul-79	9.69%	Jul-84	9.00%	Jul-89	7.00%
Aug-74	8.00%	Aug-79	10.24%	Aug-84	9.00%	Aug-89	7.00%
Sep-74	8.00%	Sep-79	10.70%	Sep-84	9.00%	Sep-89	7.00%
Oct-74	8.00%	Oct-79	11.77%	Oct-84	9.00%	Oct-89	7.00%
Nov-74	8.00%	Nov-79	12.00%	Nov-84	8.83%	Nov-89	7.00%

Jan-90	7.00%	Jan-95	4.75%	Jan-00	5.00%
Feb-90	7.00%	Feb-95	5.25%	Feb-00	5.24%
Mar-90	7.00%	Mar-95	5.25%	Mar-00	5.34%
Apr-90	7.00%	Apr-95	5.25%	Apr-00	5.50%
May-90	7.00%	May-95	5.25%	May-00	5.71%
Jun-90	7.00%	Jun-95	5.25%	Jun-00	6.00%
Jul-90	7.00%	Jul-95	5.25%	Jul-00	6.00%
Aug-90	7.00%	Aug-95	5.25%	Aug-00	6.00%
Sep-90	7.00%	Sep-95	5.25%	Sep-00	6.00%
Oct-90	7.00%	Oct-95	5.25%	Oct-00	6.00%
Nov-90	7.00%	Nov-95	5.25%	Nov-00	6.00%
Dec-90	6.79%	Dec-95	5.25%	Dec-00	6.00%
Jan-91	6.50%	Jan-96	5.24%	Jan-01	5.52%
Feb-91	6.00%	Feb-96	5.00%	Feb-01	5.00%
Mar-91	6.00%	Mar-96	5.00%	Mar-01	4.81%
Apr-91	5.98%	Apr-96	5.00%	Apr-01	4.28%
May-91	5.50%	May-96	5.00%	May-01	3.73%
Jun-91	5.50%	Jun-96	5.00%	Jun-01	3.47%
Jul-91	5.50%	Jul-96	5.00%	Jul-01	3.25%
Aug-91	5.50%	Aug-96	5.00%	Aug-01	3.16%
Sep-91	5.20%	Sep-96	5.00%	Sep-01	2.77%
Oct-91	5.00%	Oct-96	5.00%	Oct-01	2.02%
Nov-91	4.58%	Nov-96	5.00%	Nov-01	1.58%
Dec-91	4.11%	Dec-96	5.00%	Dec-01	1.33%
Jan-92	3.50%	Jan-97	5.00%	Jan-02	1.25%
Feb-92	3.50%	Feb-97	5.00%	Feb-02	1.25%
Mar-92	3.50%	Mar-97	5.00%	Mar-02	1.25%
Apr-92	3.50%	Apr-97	5.00%	Apr-02	1.25%
May-92	3.50%	May-97	5.00%	May-02	1.25%
Jun-92	3.50%	Jun-97	5.00%	Jun-02	1.25%
Jul-92	3.02%	Jul-97	5.00%	Jul-02	1.25%
Aug-92	3.00%	Aug-97	5.00%	Aug-02	1.25%
Sep-92	3.00%	Sep-97	5.00%	Sep-02	1.25%
Oct-92	3.00%	Oct-97	5.00%	Oct-02	1.25%
Nov-92	3.00%	Nov-97	5.00%	Nov-02	0.75%
Dec-92	3.00%	Dec-97	5.00%		
Jan-93	3.00%	Jan-98	5.00%		
Feb-93	3.00%	Feb-98	5.00%		
Mar-93	3.00%	Mar-98	5.00%		
Apr-93	3.00%	Apr-98	5.00%		
May-93	3.00%	May-98	5.00%		
Jun-93	3.00%	Jun-98	5.00%		
Jul-93	3.00%	Jul-98	5.00%		
Aug-93	3.00%	Aug-98	5.00%		
Sep-93	3.00%	Sep-98	5.00%		
Oct-93	3.00%	Oct-98	4.86%		
Nov-93	3.00%	Nov-98	4.63%		
Dec-93	3.00%	Dec-98	4.50%		
Jan-94	3.00%	Jan-99	4.50%		
Feb-94	3.00%	Feb-99	4.50%		
Mar-94	3.00%	Mar-99	4.50%		
Apr-94	3.00%	Apr-99	4.50%		
May-94	3.24%	May-99	4.50%		
Jun-94	3.50%	Jun-99	4.50%		
Jul-94	3.50%	Jul-99	4.50%		
Aug-94	3.76%	Aug-99	4.56%		
Sep-94	4.00%	Sep-99	4.75%		
Oct-94	4.00%	Oct-99	4.75%		
Nov-94	4.40%	Nov-99	4.86%		